

# Computer Systems Performance Analysis and Benchmarking (37-235)

Analytic Modeling  
Simulation  
Measurements / Benchmarking

Lecture/Assignments/Projects:  
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Textbook:  
Raj Jain, "The Art of Computer Systems Performance Analysis", 1991 Wiley & Sons, New York

## Topic of Today:

- The ratio game
- Mean / Median / Mode
- Clustering

# The Ratio Game

If you can't convince them, confuse them.

Truman's Law

## Based on Mathematical Fact that...

	System A	System B
Program 0	A0	B0
Program 1	A1	B1
Average	$\frac{A0 + A1}{2}$	$\frac{B0 + B1}{2}$
Relative Average	1	$\frac{B0 + B1}{A0 + A1}$

## ... is not equal

	Sys A	Sys B
Program 0	1	B0/A0
Program 1	1	B1/A1
Relative Average	1	$\frac{1}{2} \left( \frac{B0}{A0} + \frac{B1}{A1} \right)$
Relative Average	1	$\frac{1}{2} \left( \frac{A1B0 + A0B1}{A0A1} \right)$

## Graphical Version with%

TABLE 11.8 Two Tests on Two Systems

Test	System A			System B		
	Total	Pass	% Pass	Total	Pass	% Pass
1	300	60	20	32	8	25
2	50	2	4	500	40	8
	350	62	20.6	532	48	9

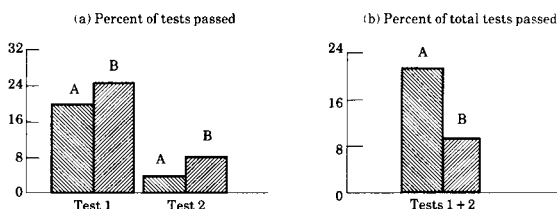


FIGURE 11.1 Ratio games with percentages.

## Consequences/Strategies

- If one system is better on all benchmarks... no contradictions.
- Even if one system is better on all benchmarks... ratio game leads to better relative number
- If one system is better in some cases and worse in other cases contradictory conclusions can be drawn sometimes.
- If the metric is LB (lower better) then use your favorite system as a base.
- If the metric is HB (higher better) then use your opponents system as a base.
- Benchmarks that perform better should be elongated, those that perform worse should be shortened.
- Remember: Taking an average of a ratio is not a correct way to analyse data.

# A mathematical Analysis.

TABLE 11.10 Derivation of the Rules

Raw Data			With A as a Base			With B as a Base		
Benchmark	System		Benchmark	System		Benchmark	System	
	A	B		A	B		A	B
I	a	ax	I	1	x	I	$\frac{1}{x}$	1
J	b	by	J	1	y	J	$\frac{1}{y}$	1
Average	$\frac{a+b}{2}$	$\frac{ax+by}{2}$		1	$\frac{x+y}{2}$		$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y}\right)$	1

Using raw data, System A is better if and only if:

$$\frac{a+b}{2} > \frac{ax+by}{2} \text{ or } y < -\frac{a}{b}x + \frac{a+b}{b}$$

Using System A as a base, System A is better if and only if:

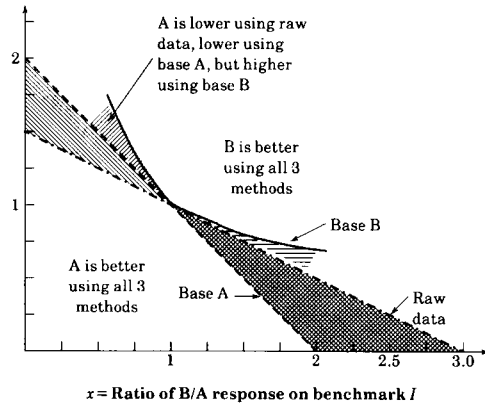
$$\frac{x+y}{2} < 1 \text{ or } y < 2-x$$

Using raw data, System A is better if and only if:

$$\frac{1}{2}\left(\frac{1}{x} + \frac{1}{y}\right) > 1 \text{ or } y < \frac{x}{2x-1}$$

# A mathematical Analysis

Ratio of B/A response on benchmark J



Strategies for ratio games. Contradictory conclusions can be obtained using different methods only in shaded regions.

## What is wrong in those games?

- Can't take the mean value of ratios!

How can we fix a good analysis

Do your homework in statistics...

Rest of this lecture:

- Index - where to look it up
- Walk through English terminology
- Recipes
- No proofs - no derivations

## Summarizing Measured Data

- Independent Events
- Random Variable
- Cumulative Distribution Function (cdf)

$$F_x(a) = P(x \leq a)$$

- Probability Density Function (pdf)

$$f(x) = \frac{dF(x)}{dx}$$

$$P(x_1 < x \leq x_2) = F(x_2) - F(x_1) = \int_{x_1}^{x_2} f(x) dx$$

- Probability Mass Function (pmf)

$$f(x_i) = p_i$$

$$P(x_1 < x \leq x_2) = F(x_2) - F(x_1) = \sum_{x_1 < x_i < x_2(1)} p_i$$

- Mean or Expected Value

$$\text{Mean} = E(x) = \sum_{i=1}^n p_i x_i = \int_{-\infty}^{\infty} x f(x) dx$$

- Variance

$$\text{Var}(x) = E[(x - \mu)^2] = \sum_{i=1}^n p_i (x_i - \mu)^2$$

$$\int_{-\infty}^{\infty} (x_i - \mu)^2 f(x) dx$$

- Standard Deviation

$$\sigma = \sqrt{\text{Var}(x)}$$

- Coefficient of Variation

$$\text{C.O.V.} = \frac{\text{std.dev.}}{\text{mean}} = \frac{\sigma}{\mu}$$

- Covariance

$$\text{Cov}(x,y) = E[(x - \mu_x)(y - \mu_y)] = E[xy] - E[x]E[y]$$

- Covariance Symbols

$$\text{Cov}(x,y) = \sigma_{xy}^2$$

- Correlation Coefficient

$$\text{Correlation}(x,y) = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

- Mean and Variance of Sums

see Formula in Book

- Quantile/Percentile

$$P(x \leq x_\alpha) = F(x_\alpha) = \alpha$$

- Median

The 50 percentile or 0.5 quantile

- Mode

Most likely value, that is  $x_i$ , that has the highest probability  $p_i$  or max of  $\text{pdf}(x_i)$ .

- Normal Distribution  $N(\mu, \sigma)$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Standard Normal Distribution  $N(0,1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

- $\alpha$ -quantile of Standard Normal Distrib.

$$z_\alpha = z \sim N(0, 1)$$

$$P\left(\frac{x - \mu}{\sigma} \leq z_\alpha\right) = \alpha$$

see tables at the end of the book

### Central Limit Theorem

- The sum of a large number of independent observations from any distribution tends to have a normal distribution.

- The sum of a normal variate is a normal variate

## Summarizing Data by a Single Number

### Averages or (indices of central tendencies):

- Sample mean
- Sample median
- Sample mode

### Selecting among them:

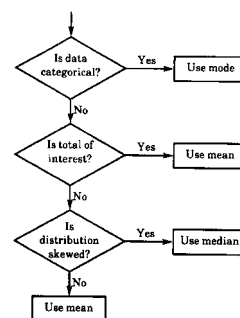


FIGURE 12.2 Selecting among the mean, median, and mode.

## Examples:

Mode: Most used resource in a system.

Mean: Interarrival time of packets.

Median: Load on a computer.

Median: Average configuration (number of devices, memory size...).

## Abuses of arithmetic mean

- Significantly different values
- Skewed distribution
- Multiplying means of dependent variables.

and once more....

- Taking means of ratio with different bases

## Other means

### Geometric mean

$$\dot{x} = \sqrt[n]{\prod_{i=1}^n x_i}$$

- works also fine for ratios
- equal to  $\exp[\text{arithmetic mean of } \log[x_i]]$ .
- very commonly used in benchmarks.

### Harmonic mean

$$\dot{x} = \frac{n}{1/x_1 + 1/x_2 + \dots + 1/x_n}$$

- works when  $1/x$  are cumulative.
- work well for rates:
  - MIPS
  - MByte/s
  - MFlops

## Summarizing Variability

### Variability is specified the following measures or indices of dispersion

- Range - [min...max]
- Variance or standard deviation
- 10 and 90-percentile
- semi-interquartile range
- mean absolute deviation

## Discussion of Variability Indices

- range  
extremely unstable, one outlier and you are gone
- sample variance and sample std.dev.

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

only  $n-1$  independent differences  $(x_i - \bar{x})$ , degree of freedom  $n-1$ .

- Variances and sample std.dev. are absolute measures.
- Variance has square as a unit.
- C.O.V is normalized by the mean and better
- C.O.V of 5 is bad, C.O.V.of 0.2 (or 20%) is good.

## Specifying quantiles

**Definition:** How much of the distribution density is within a certain range.

- 5% - 95% is about equivalent to range.
- Decile, quartiles... fixed quantiles increment 0.1 or 0.25.
- Semi-Interquartile Range is difference between Q3 and Q1.
- mean absolute deviation: Sort of a Std. Dev. but with absolute value instead of square.

**Selecting index of dispersion:**

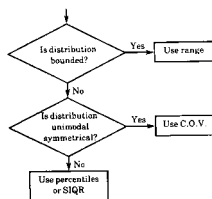


FIGURE 12.4 Selecting the correct index of dispersion.

## Determining the Distribution

**do a quantile/quantile plot**

- determine the quantile of the suspected distribution

$$q_i = F(x_i)$$

$$x_i = F^{-1}(q_i)$$

- e.g. for  $N(0,1)$  approximation as follows:

$$x_i = 4.91[q_i^{0.14} - (1 - q_i)^{0.14}]_i$$

- Plot it against the quantiles of your distribution.
- see if you can match a linear function.

Example: Modelling error for 8 predictions of a model were found to be  $y_j$ . Are errors normally distributed?

TABLE 12.5 Data for Normal Quantile-Quantile Plot Example

$i$	$q_i = \frac{i-0.5}{n}$	$y_i$	$x_i$
1	0.0625	-0.19	-1.535
2	0.1875	-0.14	-0.885
3	0.3125	-0.09	-0.487
4	0.4375	-0.04	-0.137
5	0.5625	0.04	0.137
6	0.6875	0.09	0.487
7	0.8125	0.14	0.885
8	0.9375	0.19	1.535

Result: Yes, see graphic.

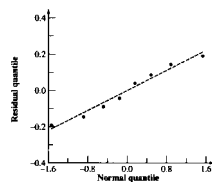


FIGURE 12.5 Normal quantile-quantile plot for the error data.

some matches on Figure 12.6 in the book

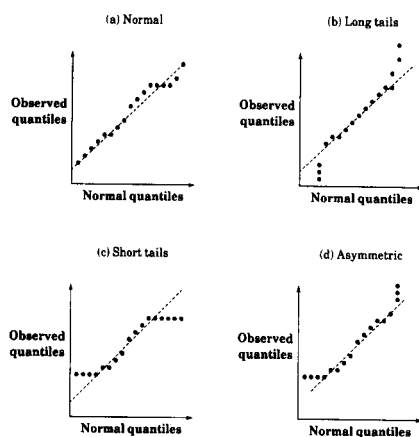


FIGURE 12.6 Interpretation of normal quantile-quantile plots.

## Clustering

- Take a sample (subset of workload components)
- Select workload parameters
- Transform parameters (e.g. to log)
- Remove outliers
- Scale data (e.g. to  $\mu=0, \sigma=1$ )
- Select a distance metric (e.g. euclid)

$$d = \sqrt{\sum_{k=1}^n (x_{ik} - x_{jk})^2}$$

- Do clustering algorithm
- Interpret results
- Change parameters (e.g. number of clusters) repeat from step transform.
- Select a representative of each cluster

## Example: Simple Spanning Tree Method

Consider workload with five components and two parameters. The CPU time and number of I/O's were measured for five programs.

**TABLE 6.6 Data for Clustering Example 6.3**

Program	CPU Time	Disk I/O
A	2	4
B	3	5
C	1	6
D	4	3
E	5	2

- For more sophisticated approaches... read an AI or IT paper for more (keyword VC dimension).

- Step 1: Consider five clusters with  $i$ th cluster consisting solely of the  $i$ th program.  
 Step 2: The centroids are  $\{2,4\}$ ,  $\{3,5\}$ ,  $\{1,6\}$ ,  $\{4,3\}$ , and  $\{5,2\}$ . These are shown by the five points in Figure 6.6.  
 Step 3: Using the Euclidean distance measure, the distance matrix is

Program	Program				
	A	B	C	D	E
A	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$
B		0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$
C			0	$\sqrt{18}$	$\sqrt{32}$
D				0	$\sqrt{2}$
E					0

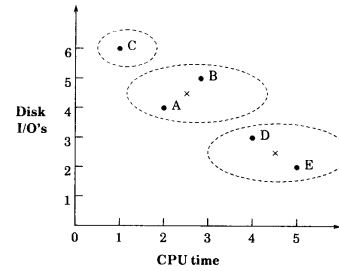


FIGURE 6.6 Clustering example.

- Step 4: The minimum intercluster distance is  $\sqrt{2}$  between A and B and between D and E. These two pairs are therefore merged.  
 Step 2: The centroid of cluster pair AB is  $\{(2+3) \div 2, (4+5) \div 2\}$ , that is,  $\{2.5, 4.5\}$ . Similarly, the centroid of pair DE is  $\{4.5, 2.5\}$ . The other centroids are the same as before.  
 Step 3: There are three clusters now, as shown in Figure 6.6, and the distance matrix is

Program	Program		
	AB	C	DE
AB	0	$\sqrt{4.5}$	$\sqrt{8}$
C		0	$\sqrt{24.5}$
DE			0

- Step 4: The minimum intercluster distance is  $\sqrt{4.5}$  between AB and C. These two clusters are therefore merged.  
 Step 2: The centroid of cluster ABC is  $\{(2+3+1) \div 3, (4+5+6) \div 3\}$ , that is,  $\{2, 5\}$ .  
 Step 3: The distance matrix is

Program	Program	
	ABC	DE
ABC	0	$\sqrt{12.5}$
DE		0

- Step 4: The minimum intercluster distance is  $\sqrt{12.5}$ . The merger of ABC and DE results in a single cluster ABCDE. □

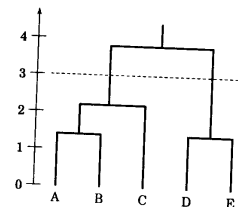


FIGURE 6.7 Dendrogram (spanning tree) for the clustering example.

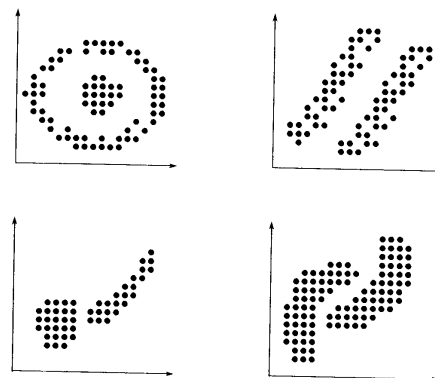


FIGURE 6.8 Problems with clustering.