Computer Systems Performance Analysis and Benchmarking (37-235)

Analytic Modeling

Simulation

Measurements / Benchmarking

Lecture by:

Prof. Thomas Stricker
Michela Taufer

Assignments/Projects:

Christian Kurmann

Textbook:

Raj Jain, "The Art of Computer Systems Performance Analysis", 1991 Wiley & Sons, New York

Topic of Today:

- **The ratio game**
- **Mean / Median / Mode**
- **Clustering**

The Ratio Game

Based on Mathematical Fact that...

	System A	System B
Program 0	A0	B0
Program 1	A1	B1
Average	$\frac{A0 + A1}{2}$	$\frac{B0 + B1}{2}$
Relative Average	1	$\frac{B0 + B1}{A0 + A1}$

... is not equal

	Sys A	Sys B
Program 0	1	B0/A0
Program 1	1	B1/A1
Relative Average	1	$\frac{1}{2}\left(\frac{B0}{A0} + \frac{B1}{A1}\right)$
Relative Average	1	$\frac{1}{2}\left(\frac{A1B0 + A0B1}{A0A1}\right)$

Graphical Version with%

TABLE 11.8 Two Tests on Two Systems

System A				System B			
Test	Total	Pass	% Pass	Test	Total	Pass	% Pass
1	300	60	20	1	32	8	25
2	50	2	4	2	500	40	8
	350	62	20.6		532	48	9

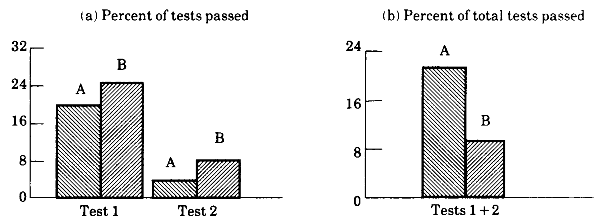


FIGURE 11.1 Ratio games with percentages.

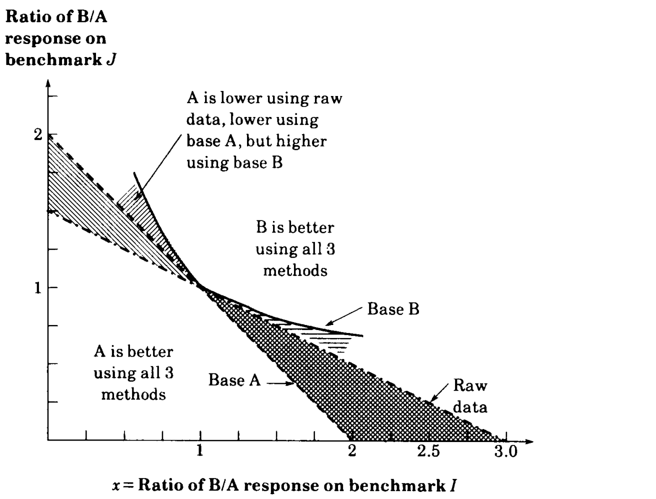
Consequences/Strategies

- If one system is better on all benchmarks... no contradictions.
- Even if one system is better on all benchmarks... ratio game leads to better relative number
- If one system is better in some cases and worse in other cases contradictory conclusions can be drawn sometimes.
- If the metric is LB (lower better) then use your favorite system as a base.
- If the metric is HB (higher better) then use your opponents system as a base.
- Benchmarks that perform better should be elongated, those that perform worse should be shortened.

A mathematical Analysis

see Derivation 11.1 in the book...

... or study Figure 11.2



Strategies for ratio games. Contradictory conclusions can be obtained using different methods only in shaded regions.

What is wrong in those games?

- Can't take the mean value of ratios!

How can we fix a good analysis

Do your homework in statistics...

Rest of this lecture:

- Index - where to look it up
- Walk trough English terminology
- Recipes
- No proofs - no derivations

Summarizing Measured Data

- Independent Events
- Random Variable
- Cumulative Distribution Function (cdf)

$$F_x(a) = P(x \le a)$$

- Probability Density Function (pdf)

$$f(x) = \frac{dF(x)}{dx}$$

$$P(x_1 < x \le x_2) = F(x_2) - F(x_1) = \int_{x_2}^{x_1} f(x)dx$$

- Probability Mass Function (pmf)

$$f(x_i) = p_i$$

$$P(x_1 < x \le x_2) = F(x_2) - F(x_1) = \sum_{x_1 < x_i < x_{2(1)}} p_i$$

- Mean or Expected Value

$$\text{Mean } \mu = \text{E(x)} = \sum_{i=1}^{n} p_i x_i = \int_{-\infty}^{\infty} x f(x) dx$$

- Variance

$$\text{Var(x)} = E[(x-\mu)^2] = \sum_{i=1}^{n} p_i (x_i - \mu)^2$$

$$\int_{-\infty}^{\infty} (x_i - \mu)^2 f(x) dx$$

- Standard Deviation

$$\sigma = \sqrt{\text{Var(x)}}$$

- Coefficient of Variation

$$\text{C.O.V.} = \frac{\text{std.dev.}}{\text{mean}} = \frac{\sigma}{\mu}$$

- Covariance

- Covariance Symbols

$$\mathrm{Cov(x,y)} = \sigma_{xy}^2$$

- Correlation Coefficient

$$\mathrm{CorrCoeff(x,y)} = \frac{\sigma_{xy}^2}{\sigma_x \sigma_y}$$

- Mean and Variance of Sums

  see Formulas in book

- Quantile/Percentile

$$P(x \leq x_\alpha) = F(x_\alpha) = \alpha$$

- Median

  The 50 percentile or 0.5 quantile

- Mode

  Most likely value for x_i, that is x_i with max(p_i) or global max of pdf(x_i).

- Normal Distribution N(μ,σ)

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

- Standard Normal Distribution N(0,1)

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

- α-quantile of Standard Normal Distrib.

$$z_\alpha = z \sim N(0, 1)$$

$$P\left(\frac{x-\mu}{\sigma} \leq z_\alpha\right) = \alpha$$

  see tables at the end of the book

Central Limit Theorem

- The sum of a large number of independent observation from any distribution tens to have a normal distribution.
- The sum of a normal variate is a normal variate

Summarizing Data by a Single Number

Averages or (indices of central tendencies).

- Sample mean
- Sample median
- Sample mode

Selecting among them:

see figures and decision chart in the book

Examples:

Mode: Most used resource in a system.

Mean: Interarrival time of packets.

Median: Load on a computer.

Median: Average configuration RAM...

Abuses of arithmetic mean

- Significantly different values
- Skewed distribution
- Multiplying means of dependent variables.

and once more....

- Taking means of ratio with different bases

Other means

Geometric mean

$$\dot{x} = \sqrt[n]{\prod_{i=1}^{n} x_i}$$

- works also fine or ratios...
- equal to exp[arithmetic mean of log[x_i]]).
- very commonly used in benchmarks.

Harmonic mean

$$\ddot{x} = \frac{n}{1/x_1 + 1/x_2 + \ldots + 1/x_n}$$

- works when 1/x are cumulative.
- work well for rates:
  - MIPS
  - MByte/s
  - MFlops

Variability

or indices of dispersion by statisticians...

- Range - [min...max]
- Variance or standard deviation
- 10 and 90-percentile
- semi-interquantile range
- mean absolute deviation

Discussion of Variability Indices

- range

  extremely unstable. One outlier and you are gone.
- sample variance and sample std.dev.

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

  only n-1 independent x_i's, degree of freedom n-1.
- Variances and sample std.dev. are absolute measures.
- Variance has square as a unit.
- C.O.V is normalized by the mean and better
- C.O.V of 5 is bad, C.O.V.of 0.2 is good.

Specifying quantiles

Definition: How much of the distribution density is within a certain range.

- 5% - 95% is about equivalent to range.
- Decile, quartiles... fixed quantiles increment 0.1 or 0.25.
- Semi-Interquartile Range is difference between Q3 and Q1.
- mean absolute distribution: Sort of a Std. Dev. but with abs. value instead of square.

Selecting index of dispersion:

- see decision chart in the book.

Higher indices:

- skewedness and curtosis
- rarely used in performance evaluation

Determining the Distribution

do a quantile/quantile plot

- determine the quantile of the suspected distribution

$$q_i = F(x_i)$$
$$x_i = F^{-1}(q_i)$$

- e.g. for N(0,1)

$$x_i = 4.91[q_i^{0.14} - (1 - q_i)^{0.14}]_i$$

- Plot it against the quantiles of your distribution.
- see if you can match a linear function.

some matches on Figure 12.6 in the book.

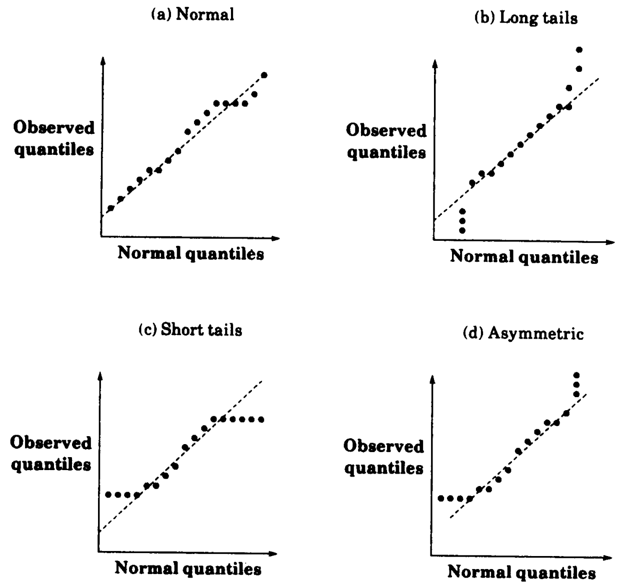


FIGURE 12.6 Interpretation of normal quantile-quantile plots.

Clustering

- Take a sample (subset of workload Components)
- Select parameters
- Transform parameters e.g. to log
- Remove outliers
- Scale data (e.g. to $\mu=0$, $\sigma=1$)
- Select a distance metric (e.g. euclid)

$$d = \sqrt{\sum_{k=1}^{n} (x_{ik} - x_{jk})^2}$$

- Do clustering algorithm
- Interpret results
- Change parameters (e.g. number of clusters) repeat from step transform.
- Select a representative of each cluster

Example:

- Simple Spanning Tree Method

TABLE 6.6 Data for Clustering Example 6.3

Program	CPU Time	Disk I/O
A	2	4
B	3	5
C	1	6
D	4	3
E	5	2

- For more sophisticated approaches... read an AI or IT paper for more (keyword VC dimension).

Step 1: Consider five clusters with ith cluster consisting solely of the ith program.

Step 2: The centroids are {2,4}, {3,5}, {1,6}, {4,3}, and {5,2}. These are shown by the five points in Figure 6.6.

Step 3: Using the Euclidean distance measure, the distance matrix is

Program	A	B	C	D	E
A	0	$\sqrt{2}$	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$
B		0	$\sqrt{5}$	$\sqrt{5}$	$\sqrt{13}$
C			0	$\sqrt{18}$	$\sqrt{32}$
D				0	$\sqrt{2}$
E					0

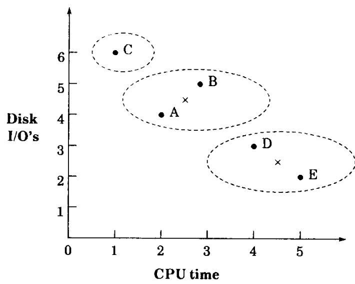


FIGURE 6.6 Clustering example.

Step 4: The minimum intercluster distance is $\sqrt{2}$ between A and B and between D and E. These two pairs are therefore merged.

Step 2: The centroid of cluster pair AB is $\{(2+3)\div 2, (4+5)\div 2\}$, that is, {2.5, 4.5}. Similarly, the centroid of pair DE is {4.5, 2.5}. The other centroids are the same as before.

Step 3: There are three clusters now, as shown in Figure 6.6, and the distance matrix is

Program	AB	C	DE
AB	0	$\sqrt{4.5}$	$\sqrt{8}$
C		0	$\sqrt{24.5}$
DE			0

Step 4: The minimum intercluster distance is $\sqrt{4.5}$ between AB and C. These two clusters are therefore merged.

Step 2: The centroid of cluster ABC is $\{(2+3+1)\div 3, (4+5+6)\div 3\}$, that is, {2,5}.

Step 3: The distance matrix is

Program	ABC	DE
ABC	0	$\sqrt{12.5}$
DE		0

Step 4: The minimum intercluster distance is $\sqrt{12.5}$. The merger of ABC and DE results in a single cluster ABCDE. □

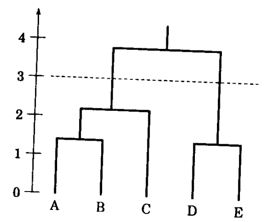


FIGURE 6.7 Dendrogram (spanning tree) for the clustering example.

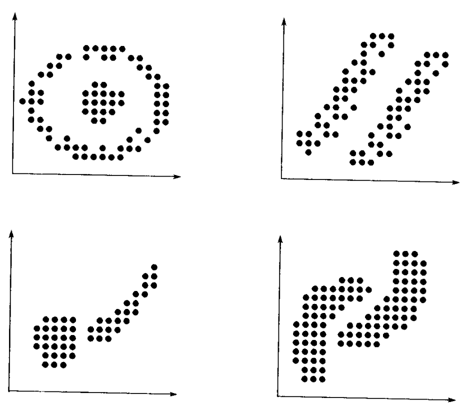


FIGURE 6.8 Problems with clustering.